J. Fluid Mech. (1967), *vol.* 30, *part* 4, *pp.* 813–821
Printed in Great Britain

Measurements of drag in a conducting fluid with an aligned field and large interaction parameter

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(Received 23 June 1967)

The drag of spheres and disks has been measured in a flow of liquid sodium within an aligned magnetic field. A slightly viscous, strong field limit is discussed and explored experimentally. In this limit the drag coefficient was found to have an asymptotic dependence proportional to the square root of the interaction parameter, the ratio of magnetic to inertial force, independent of body shape. A physical model is presented along with preliminary verification of its basic characteristics.

1. Introduction

Although the problem of aligned fields magneto-fluid dynamic (MFD) flow past bodies has received much attention from applied mathematicians in the last ten years, there has been a general lack of emphasis on cases which can be realized in the laboratory. This situation has been partly due to analytical difficulties and partly due to the limited interest and success of the experimenters themselves. The steady dimensionless equations for a viscous incompressible conducting flow are

$$\mathbf{V}.\nabla\mathbf{V}+\nabla P = N(\mathbf{V}\times\mathbf{B})\times\mathbf{B}+Re^{-1}\nabla^2\mathbf{V},$$
$$\nabla\times\mathbf{B} = Rm(\mathbf{V}\times\mathbf{B}),$$
$$\nabla.\mathbf{B} = \nabla.\mathbf{V} = 0,$$

where $V = V'/V_0$, $P = (P'-P_0)/V_0^2\rho$, $x = x'/d$, $B = B'/B_0$, and $N = B_0^2\sigma d/V_0$, the interaction parameter; $Re = V_0 d/\nu$, the Reynolds number; and $Rm = \sigma\mu V_0 d$, the magnetic Reynolds number. Although these nonlinear coupled equations have not been solved for arbitrary values of the three parameters, various limiting cases have been considered in great detail.

Gourdine (1961) has studied the coupling of the equations under arbitrary conditions by considering an Oseen-type approximation. This yields two rotational velocity perturbation modes $U_{1,2}$ satisfying $(\nabla^2-\lambda_{1,2})U_{1,2} = 0$, where $\lambda_{1,2}$ are given by

$$\frac{Re}{2}\left[1+\frac{Rm}{Re}\pm\left\{\left(1+\frac{Rm}{Re}\right)^2-4\frac{Rm}{Re}\left(1-\frac{N}{Rm}\right)\right\}^{\frac{1}{2}}\right].$$

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Table 1 shows the approximate values of the parameters which appear in $\lambda_{1,2}$ under achievable laboratory conditions. One finds that Rm/Re, a material property, is always much less than one, and that N/Re, typically quite small, can be made ~ 1 in an actual experiment only by reducing the velocity by at least 1 order of magnitude below the value suggested here. The resulting pressures and drag would then be of such a low level to make experiments extremely difficult.

	Na (135 °C)	NaK (20 °C)	Hg (20 °C)
Rm	0·07	0·02	0·006
Re	8,000	6,000	40,000
N	200	70	2·0

$V_0 = 0\cdot50\,\text{m/sec}, \quad B_0 = 1\,\text{Wb/m}^2, \quad d = 0\cdot01\,\text{m}$

TABLE 1

The one ratio that can be varied over a wide range is $N/Rm = B_0^2/\rho\mu V_0^2$, the square of the ratio of the Alfvén speed to the free-stream speed. For $N/Rm \ll 1$, this general class of super-Alfvénic flows yields $\lambda_{1,2} = Rm, Re$. Both modes give downstream wakes, and, since $Rm < 1$ in typical experiments, the dominant mode is the ordinary viscous wake. One expects that in such a case the flow would only be slightly modified, if at all, by MFD effects, and is of little interest.

On the other hand, for $N/Rm \gg 1$, sub-Alfvénic, strong effects do occur and fall into two classes depending on the value of N/Re. For

$$N/Re \gg 1, \quad \lambda_{1,2} = -(NRe)^{\frac{1}{2}}, \quad +(NRe)^{\frac{1}{2}},$$

symmetric magneto-viscous wakes exist. The other class of sub-Alfvénic flows, $N/Re \ll 1$, is easily realizable and yields $\lambda_{1,2} = -N, +Re$. If $N \ll 1$ the forward wake is very diffuse, the perturbations from potential flow upstream are small, and regular perturbation methods have been successful in treating such problems. On the other hand, for $N \gg 1$, large perturbations arise within the concentrated forward wake. The present work is concerned principally with this asymptotic case for which neither a satisfactory theoretical nor experimental treatment exists.

If we further consider the limit $N \to \infty$, $Re \to \infty$, the Oseen analysis yields simply $\partial U_{1,2}/\partial x = 0$. The statement then that perturbation quantities are independent of the co-ordinate along the applied field is analogous to that of the Taylor–Proudman theorem which applies to the case of strongly rotating, slightly viscous flow. The analogy has in the past been limited to the case of infinite conductivity (Chandrasekhar 1961), but the above limit for finite conductivity and strong field yields the same conclusion. Therefore one would expect stagnant slugs similar to the 'Taylor Column' to form.

Liepmann, Hoult & Ahlstrom (1960) described unsuccessful attempts to measure the drag of freely rising spheres in a tank of mercury within a solenoid. Ahlstrom (1963), using the same facility, demonstrated the existence of the forward wake, but found experimentally that for the chosen blockage ratio ($d/D = \frac{1}{6}$), wall effects were dominant for $N \lesssim 1$. Maxworthy (1962) measured the drag of spheres falling within a similar facility using liquid sodium, finding a marked increase in drag for strong fields. Motz (1965) measured the drag of a non-conducting sphere undergoing small-amplitude oscillations in a container of mercury within a solenoid for the case $N < 1$ and found good agreement with a linear inviscid analysis.

The goal of the present work was to measure the drag of bodies under a wider range of N than had been previously considered. To do so a strain gauge drag balance and wire suspension system were used in the Jet Propulsion Laboratory sodium flow facility.

2. Experiment

The sodium flow facility, which is similar to an ordinary water tunnel in its basic characteristics, has been described in detail by Maxworthy (1961). Liquid sodium is circulated in a closed loop which is maintained at a temperature of 135 °C and the flow rate is measured with a conventional crossed field flow meter. The test section (figure 1) is surrounded by an oil-cooled solenoid which provides a field uniform to within 3 % over 70 % of its length. An entrance nozzle is employed to minimize the currents and vorticity which are induced as the fluid enters the fringe field. The flow field within the test section has been studied for various flow and field conditions using a conventional Pitot tube technique. T. Maxworthy (private communication) has shown that the flow field is sufficiently uniform to study flows past bodies within the range of conditions employed here, $0{\cdot}9\,\text{cm/sec} < V_0 < 13\,\text{m/sec}$ and $B_0 < 0{\cdot}70\,\text{Wb/m}^2$.

The test bodies were suspended from three tungsten–rhenium wires which pass through $\frac{1}{16}$ in. holes in the test section wall into a chamber where they are attached to three aluminium beams. A temperature-compensated strain gauge bridge composed of matched encapsulated foil gauges is attached to one of the beams. The chamber is packed with silicone grease to damp beam oscillations and to protect the gauges. The balance is dead weight calibrated before and after each run at the operating temperature, which is monitored by a thermocouple attached to one beam.

The material properties for sodium at the operating temperature were found by interpolation from data appearing in the *Liquid Metals Handbook*.† They are as follows:

$$\rho = 0{\cdot}918\times10^3\,\text{kg/m}^3,\quad \sigma = 0{\cdot}911\times10^7\,\text{mho/m},\quad \rho\nu = 0{\cdot}575\times10^{-3}\,\text{kg/m.sec}.$$

This gave the following ranges of the three parameters:

$$0 < N < 80,\quad 10^4 < Re < 25\times10^4,\quad 0{\cdot}10 < Rm < 2{\cdot}50.$$

The following configurations were utilized: 0·500 in. sphere, 0·500 in. disk and 0·750 in. sphere with 0·010 in. suspension wires; 0·250 in. sphere and 0·500 in. sphere with 0·005 in. wires.

In calculating the body drag coefficient the wire drag must be subtracted from the total force measured. Since N, based on wire diameter, was less than unity

† Third edition, Atomic Energy Commission (1955).

throughout the experiments, we assumed that wire drag coefficient was unaffected by the presence of the magnetic field and used values for drag coefficient given in Schlichting (1960). The drag balance, although designed as a steady device, also served to indicate the unsteadiness of the wake. The unsteady component of the force was amplified and recorded on an oscillograph but no attempt was made to separate the unsteady drag and lift, or to quantify this measurement.

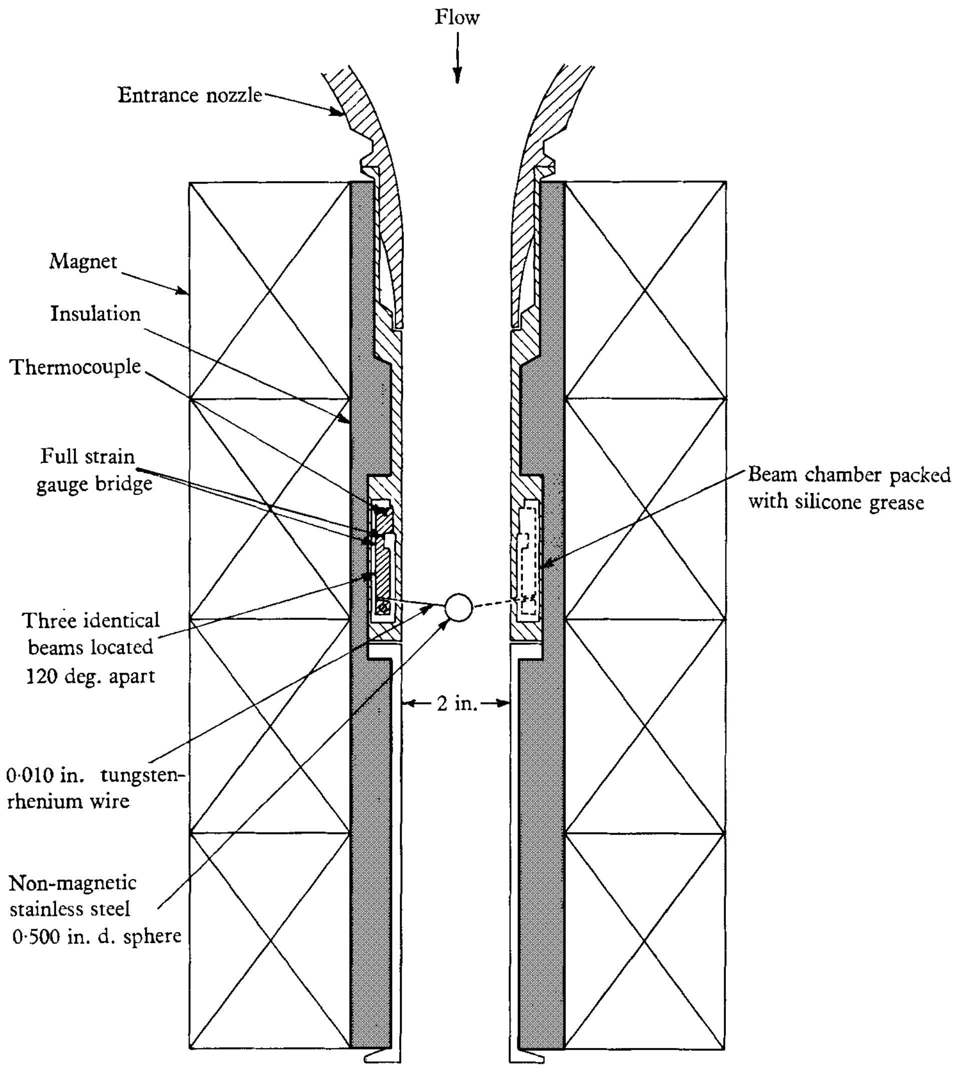

Figure 1. Drag balance test section.

3. Results and discussion

Figure 2 shows the drag coefficient, C_D, of the spheres plotted as a function of the interaction parameter, N. It appears that for a given d/D, where D is tunnel diameter, C_D is only a function of N and that weak and strong field régimes exist. For $0 < N < 1$, $N/Rm \lesssim 1$, the C_D remains relatively unchanged, and the points at the left of each graph indicate the scatter in the measured value of C_D for $B = 0$. The no-field C_D increases with d/D, because of blockage, and the extrapolated value to $d/D = 0, C_D = 0{\cdot}40$ agrees with the experimental data of previous investigators for the range $10^4 < Re < 25 \times 10^4$. One can conclude that ordinary separated flow dominates the small N régime and that magnetic forces have a negligible effect on drag.

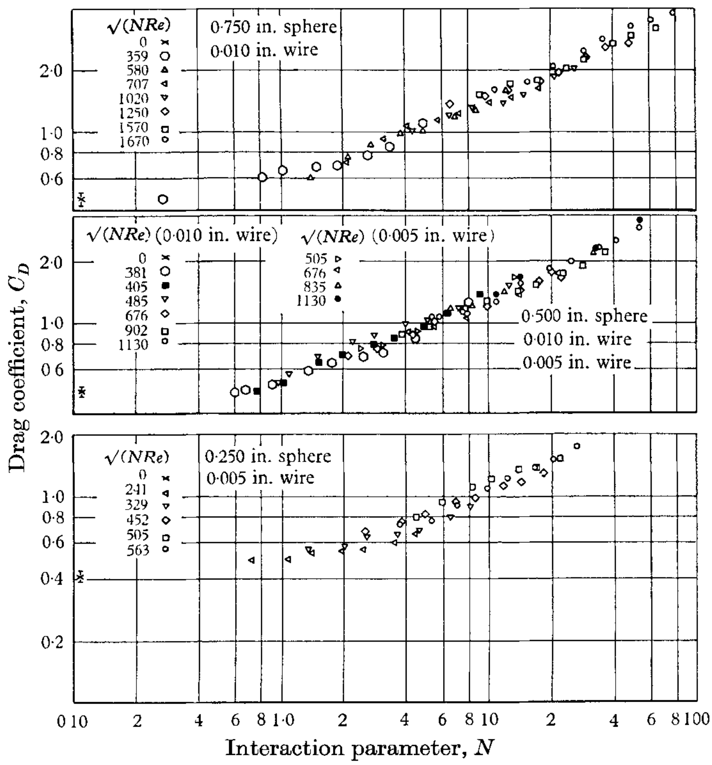

FIGURE 2. Drag coefficient *vs.* interaction parameter for spheres.

One should note that this weak field régime is not well defined in terms of the conditions given earlier since both N and N/Rm are of the order 1 or smaller. It is desirable to investigate the behaviour of C_D for N/Rm large as N alone varies through 1, and this has been done by Motz (1965), who found that C_D was 10 % lower than the potential flow prediction for $N \sim 0{\cdot}10$.

The intermediate range, $1 < N < 10$, $N/Rm \gtrsim 1$, showed no simple power law dependence on N and is likely to be characterized by both MFD and ordinary separated flow effects being important. Although this range may contain several interesting aspects, it is doubtful that any simple physical picture will be suitable in understanding them.

The strong field régime, $N/Rm \gg 1$, $N > 10$, is well defined and gave C_D proportional to $N^{\frac{1}{2}}$. The effect of blockage is observed and, if one assumes that it has the simple effect of increasing the effective velocity which should be used in calculating the appropriate non-dimensional parameters, then this predicts that $C_D \propto (1-d^2/D^2)^{-\frac{3}{2}} N^{\frac{1}{2}}$. The results of such a correction are shown in figure 3 applied over the full range of N. An extrapolation to $d/D = 0$ for $N > 10$ gave $C_D = 0{\cdot}33 N^{\frac{1}{2}}$ as indicated by the solid line in the figure. The suitability of a simple solid body blockage correction is limited to $N > 10$ and this strongly implies that for this range the wake spreads so slowly as not to interact with the walls. These results also imply that end effects due to the finite length of the test section are not important but would presumably play an increasingly important role as N is further increased.

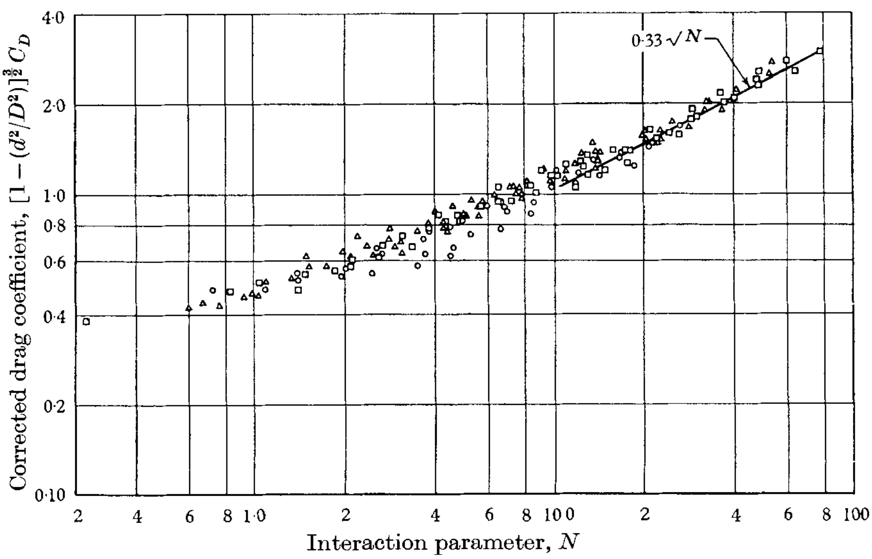

FIGURE 3. Sphere drag coefficient corrected for blockage *vs.* interaction parameter. d/D: ○, 0·125; △, 0·250; □, 0·375.

In order to investigate the effect of body shape, the drag of a 0·500 in. disk with 0·010 in. suspension wires was measured. Results are shown in figure 4 together with those for the 0·500 in. sphere. The no-field case shows a considerable blockage effect but agrees well with a partly empirical correction given by Maskell (1963), which includes the effect of the wake interacting with the tunnel wall. Again, the magnetic field has little effect for $N < 1$, but C_D does in fact decrease slightly for $1 \leqslant N \leqslant 10$. The field in this intermediate range seems to have the effect of reducing the wake blockage without appreciably affecting the overall drag. For $N > 10$, the disk C_D is very close to that of the sphere and the same asymptotic dependence is reached for $N > 20$. This result is essential in determining that the asymptotic behaviour can only exist for $N > 10$.

Although no attempt was made to obtain quantitative information concerning wake stability, certain observations of the unsteady force component were made. They were indications that relatively weak fields ($\sim 0{\cdot}30\ \mathrm{Wb/m^2}$) were able to damp completely the dominant frequencies which had existed without an applied field. Similar results were obtained by Maxworthy (1962), who noted non-oscillatory wake behaviour for $N \sim 0{\cdot}5$. On the other hand, we observed unsteadiness again for fields greater than 0·50 Wb/m² and the dominant frequency was seen to decrease monotonically as the magnetic field was further increased. The Strouhal number, $S = fd/V_0$, where f is the frequency of these oscillations, was quite small and dropped from a value of approximately 0·09 at 0·50 Wb/m² to 0·03 at 0·70 Wb/m².

These low-frequency oscillations appeared when the steady drag and presumably the wake structure were considerably different from that with no field. It is possible that long wavelength disturbances were selectively amplified in the wakes, but detailed measurements of fluctuating velocity or magnetic field

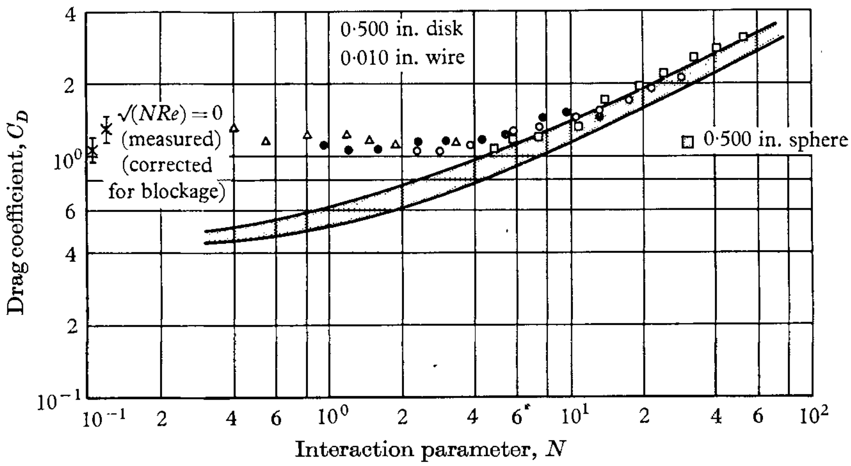

FIGURE 4. Disk drag coefficient *vs*. interaction parameter. $(NRe)^{\frac{1}{2}}$: ×, 0; △, 297; ●, 505; ○, 789; □, 1127.

perturbation would be needed to understand this result. There was a definite trend toward lower values of S with increasing field, but no obvious dependence of S was found on N itself. Since C_D depended solely on N, for a given body, it was concluded that the unsteady component was not a controlling factor in determining the steady drag.

A simple drag law, $C_D \propto N^{\frac{1}{2}}$ independent of body shape, was found under the limiting conditions $N/Rm \gg 1$, $N/Re \ll 1$ and $N \gg 1$, $Re \gg 1$. A slightly viscous, strong field limit should be amenable to an analytical solution and, although this has not been accomplished, a physical model which involves steady flow and contains the above result will be presented.

One can understand the fact that C_D is independent of body shape and the correlation with a simple solid body blockage correction if one considers the body to be surrounded by slender, relatively stagnant, constant pressure regions which are themselves separated from the outer flow by thin dissipation layers. Childress (1963) suggested a model which required the existence of layers the order of $N^{-\frac{1}{2}}$ in thickness. He discusses a singular perturbation technique for the treatment of such layers and finds that the pressure and consequently drag coefficient is less than or of the order of 1·0. Such a model is not unique and other scaling laws could apply which would agree with the experimental result that $C_D \propto N^{\frac{1}{2}}$.

In order for C_D to increase with N above a value of 1·0, there must be an increasingly large suction on the base of the body. In order to realize this, one must consider the consequences of an expression derived by Tamada (1962) for the behaviour of the Bernoulli function, $H = P + \frac{1}{2}V^2$. By taking the dot product of V with the momentum equation, one finds, for the inviscid limit, $\mathbf{V}.\nabla H = -N(V \times B)^2$. The Bernoulli function is therefore found to be a non-increasing function along streamlines. The maximum pressure will therefore be stagnation pressure and must decrease along streamlines which are not parallel with the field. The measured values of C_D for large N can consequently be explained only by a large negative pressure within the downstream slug. The asymptotic behaviour of $C_D \propto N^{\frac{1}{2}}$ then requires that the base pressure be proportional to $-N^{\frac{1}{2}}$.

Yonas (1966) has proposed that the Lorentz force within dissipation layers the order of $N^{-\frac{1}{4}}$ in thickness can support the pressure gradient linking the low-pressure slug and the outer undisturbed flow. One can write corresponding boundary-layer equations finding in the limit $N \rightarrow \infty$ that the x and y components of the momentum equation become, respectively, $\partial P/\partial x = 0$, $\partial P/\partial y = -NVy$. The solution gives the pressure to within two undetermined functions of y and, although a higher-order expansion was suggested to remove this indeterminacy, this has yet to be done. It is also conceivable that the largest part of the total dissipation would occur within these layers, permitting one to calculate the drag from a simple energy balance. A solution incorporating the aspects of such a model, or an experimental verification using pressure and magnetic probes, is required.

Further verification of this 'slug' behaviour has been found in a separate experiment using a tank of sodium potassium eutectic within a uniform field. For N as large as 600, a disk was moved within the tank towards the free surface and with the body roughly 10 diameters from it, an upwelling only within the area subtended by the body was noted. A downwelling of the same size was observed with the disk moving away from the surface. This demonstrated the usefulness of a tow tank facility for the study of large N flows, the existence of weakly damped Alfvén waves and of a concentrated forward wake.†

4. Conclusion

A measurement of the drag coefficient of spheres and disks over a wide range of N has been carried out. A simple drag law, $C_D \propto N^{\frac{1}{2}}$ independent of body shape was found under the limiting conditions $N/Rm \gg 1$, $N/Re \ll 1$, $N \gg 1$, $Re \gg 1$. A physical model explaining such a result was presented which involved slender, relatively stagnant regions which increase in length as N is increased, and preliminary observations of this phenomenon were described.

This work, carried out at the Jet Propulsion Laboratory, California Institute of Technology, was supported under Contract No. NAS 7-100 and submitted in

† A similar observational technique was reported by Liepmann *et al.* (1960) which showed an upstream perturbation approximately 10 diameters wide for sub-Alfvénic flow, but $N \sim 1$.

partial fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering Science, California Institute of Technology. The guidance of Prof. H. W. Liepmann and Dr T. Maxworthy in the conducting of this research is gratefully acknowledged. The author is also indebted to E. Coury and D. Griffith, who provided invaluable assistance in the construction and carrying out of the experiments.

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